

Medians and Altitudes of Triangles

Lesson Objective INBAT apply thms $\approx$ centroids; find locations of centroids and orthocenters in coordinate plane

Median: A median of a triangle is a segment from a vertex to the midpoint of the opposite side

The three medians of the triangle are *concurrent*. This point of concurrency is called the centroid.

The ratio of **larger** : **smaller** segments within the median is 2:1.

What would the ratio of the larger segment of the median to the ENTIRE median be?

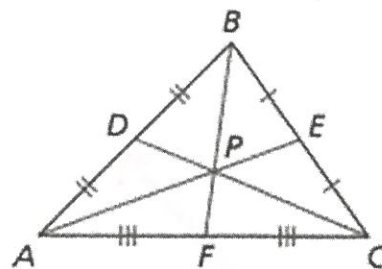
2:3

Centroid Theorem

The centroid of a triangle is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

The medians of $\triangle ABC$ meet at point P, and...

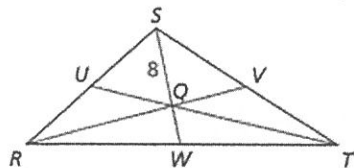
$$AP = \frac{2}{3}(AE); BP = \frac{2}{3}(BF); CP = \frac{2}{3}(CD)$$



The centroid is also called the "balance point" or "center of gravity"

Example:

1. In $\triangle RST$, $RV = 9$ and $QU = 2.4$. Find RQ and TU .



$$RQ = \frac{2}{3}RV = \frac{2}{3}(9) \quad \boxed{RQ = 6}$$

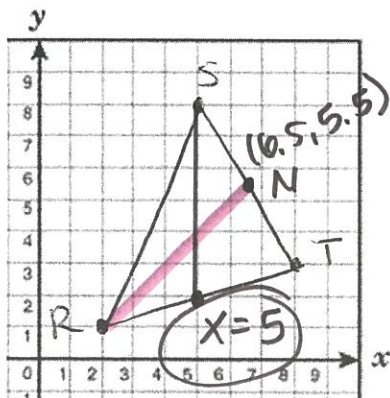
$QU: TQ$ is 1:2

$$TQ = 2(2.4) = 4.8$$

$$TU = TQ + QU = 4.8 + 2.4$$

$$\boxed{TU = 7.2}$$

2. Find the coordinates of the centroid of $\triangle RST$ where $R(2, 1)$, $S(5, 8)$, $T(8, 3)$.



$$m_{RN} = \frac{5.5 - 1}{6.5 - 2} = \frac{4.5}{4.5} = 1$$

$$y - 1 = 1(x - 2)$$

$$y - 1 = x - 2$$

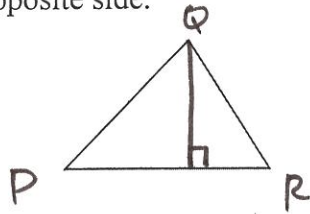
$$\boxed{y = x - 1}$$

$$y = 5 - 1$$

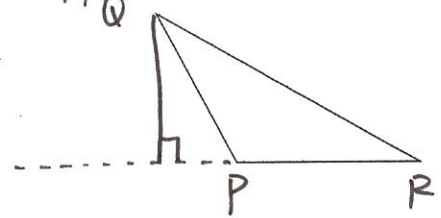
$$y = 4$$

$$\boxed{(5, 4)}$$

Altitude: An altitude of a triangle is the $\perp$ segment from a vertex to the opposite side OR the line that contains the opposite side.



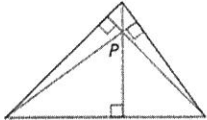
Altitude from Q to $\overleftrightarrow{PR}$



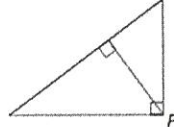
The lines containing the altitudes of a triangle are concurrent.

This point of concurrency is the orthocenter of the triangle.

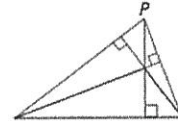
The location of the orthocenter (P, in the diagram below) depends on the type of triangle.



inside Δ
(acute Δ)



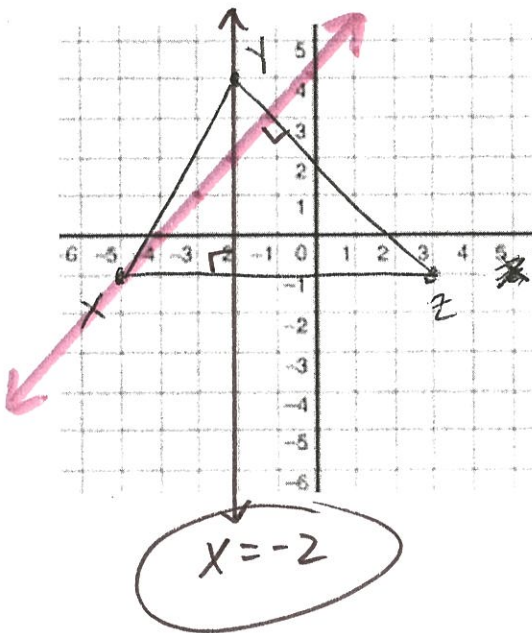
on vertex of rt. $\angle$
(right Δ)



outside Δ
(obtuse Δ)

Example:

3. Find the coordinates of the orthocenter of ΔXYZ with vertices $X(-5, -1)$, $Y(-2, 4)$ and $Z(3, -1)$



altitude from X

$$m_{YZ} = -1$$

$$m_{\perp} = 1$$

$$y - (-1) = 1(x - (-5))$$

$$y + 1 = x + 5$$

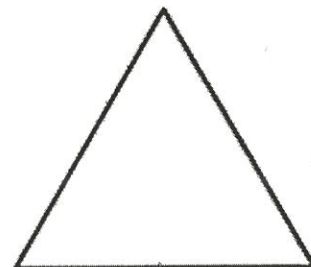
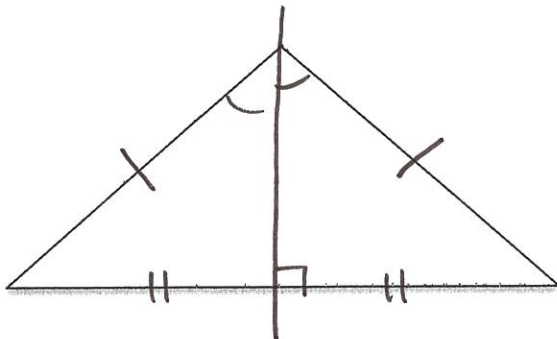
$$y = x + 4$$

$$y = -2 + 4$$

$$y = 2$$

$$(-2, 2)$$

In an isosceles Δ , the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment. In an equilateral Δ , this is true for any vertex.



See pg 323 for summary chart for circumcenter, incenter, centroid and orthocenter